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# Optimal hedging using cointegration

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Cointegration is a time-series modelling methodology that has many applications to financial markets. When spreads are mean reverting, prices are cointegrated. Then a multivariate model will provide further insight into the price equilibria and returns causalities within the system. Spot-futures arbitrage, yield curve modelling, index tracking and spread trading are some of the applications of cointegration that are reviewed in this paper. With the demand for new quantitative approaches to active investment management strategies there is considerable interest in cointegration theory. This paper presents a model of cointegrated international equity portfolios which is currently used for hedging within the European, Asian and Far East countries.

**Keywords:** cointegration; hedging; index tracking; financial markets

## 1. Introduction

Since the seminal work of Engle & Granger (1987), cointegration has become the prevalent tool of time-series econometrics. Every modern econometrics text covers the statistical theory necessary to master the practical application of cointegration, Campbell *et al.* (1997), Hamilton (1994) and Hendry (1995, 1986) being amongst the best sources. Cointegration has emerged as a powerful technique for investigating common trends in multivariate time-series, and provides a sound methodology for modelling both long-run and short-run dynamics in a system.

Although models of cointegrated financial time-series are now relatively commonplace in the literature, their importance has, until very recently, been mainly theoretical. This is because the traditional starting point for portfolio risk management in practice is a correlation analysis of returns, whereas cointegration is based on the raw price, rate or yield data. In standard risk-return models these price data are differenced before the analysis is even begun, and differencing removes *a priori* any long-term trends in the data. Of course, these trends are implicit in the returns data, but any decision based on long-term common trends in the price data is excluded in standard risk-return modelling.

Cointegration and correlation are related, but different concepts. High correlation of returns does not necessarily imply high cointegration in prices. An example is given in figure 1, with a 10 year daily series on US dollar spot exchange rates of the German Mark (DEM) and the Dutch Guilder (NLG) from 1982 to 1992. Their returns are very highly correlated: the correlation coefficient is approximately 0.98 (figure 1*a*). So also do the rates move together over long periods of time, and they appear to be cointegrated (figure 1*b*). Now suppose that we add a very small daily incremental return of, say, 0.0002 to NLG. The returns on this NLG ‘plus’ and the

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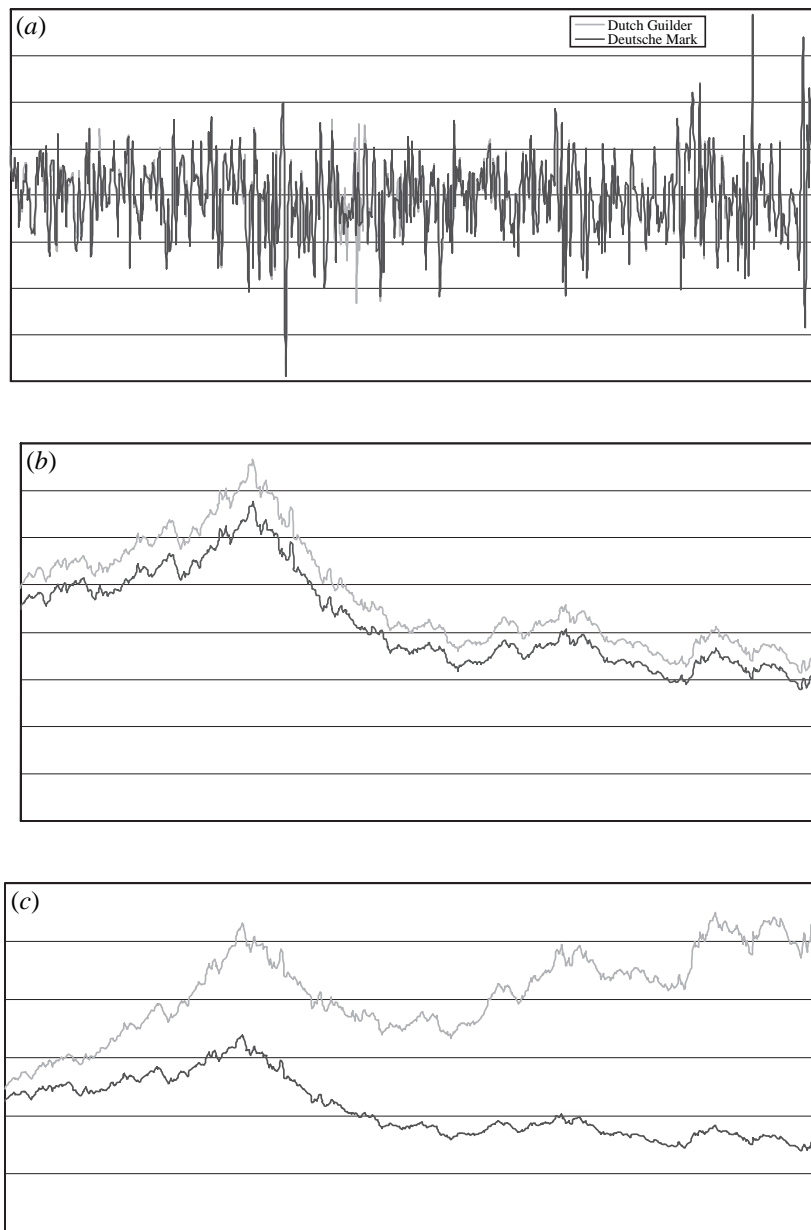


Figure 1. (a) NLG and DEM returns. (b) NLG and DEM rates. (c) NLG 'plus' versus DEM.

DEM still have correlation of 0.98, but the price series diverge and they are not cointegrated (figure 1c).<sup>†</sup>

Correlation reflects co-movements in returns, which are liable to great instabilities over time. It is intrinsically a short-run measure, and correlation-based hedging strategies commonly require frequent rebalancing. On the other hand, cointegration

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measures long-run co-movements in prices, which may occur even through periods when static correlations appear low. Therefore, hedging methodologies based on cointegrated financial assets may be more effective in the long term.

In summary, investment management strategies that are based only on volatility and correlation of returns cannot guarantee long-term performance. There is no mechanism to ensure the reversion of the hedge to the underlying, and nothing to prevent the tracking error from behaving in the unpredictable manner of a random walk. Since high correlation alone is not sufficient to ensure the long-term performance of hedges, there is a need to augment standard risk-return modelling methodologies to take account of common long-term trends in prices. This is exactly what cointegration provides. It extends the traditional models to include a preliminary stage in which the multivariate price data are analysed, and then augments the correlation analysis to include the dynamics and causal flows between returns.

This paper begins with a short survey of the theory and application of cointegration to pricing, hedging and trading portfolios of financial assets. Section 2 covers the basic mathematics necessary to understand the main concepts and § 3 reviews some of the literature on cointegration in finance. We emphasize the implications of cointegration for hedging strategies and draw attention to areas where mis-pricing and over-hedging can occur if cointegration is ignored. Examples of cointegrated financial assets abound: spot and futures; bonds of different maturities, or even between countries, in fact anywhere where spreads are mean reverting; related commodities (when carry costs are well behaved); and equities within an index, or between international indices. It is this last example that is studied in detail. Asset managers are developing increasingly quantitative approaches and with the need for new, active management strategies there is considerable interest in cointegration at present. Section 4 presents one such model of cointegrated international equity portfolios that has been developed and extended over several years and is currently used for hedging within the EAFE countries.

## 2. The mathematics of cointegration

### (a) Basic notation and terminology

The term ‘mean reversion’ in finance is used to describe what in statistical terms is called a ‘covariance stationary’ time-series, denoted  $I(0)$ . This is a stochastic process with constant finite mean and variance, and an autocorrelation that is independent of time (depending only on the lag). Typically, financial returns are (covariance) stationary processes.

A series which is non-stationary, and only stationary after differencing a minimum of  $n$  times, is called ‘integrated of order  $n$ ’, denoted  $I(n)$ . An example of an integrated series of order 1 is a random walk

$$y_t = \alpha + y_{t-1} + \varepsilon_t,$$

where  $\alpha$  denotes the drift, and the error process  $\varepsilon_t$  is an independent and identically distributed (i.i.d.) stationary process. If financial markets are strongly efficient, then log prices are random walks:  $y_t = \log P_t$ , and  $\varepsilon_t$  denotes the return at time  $t$ . It is not uncommon to have some autocorrelation in returns, assuming they are  $I(0)$  but not i.i.d. In this case, log prices are still integrated processes, but not pure random walks (Lo & MacKinlay 1988).

When log asset price time-series are integrated, over a period of time they may have wandered virtually anywhere. There is little point in modelling them individually since the best forecast of any future value is the value today plus the drift. However, a *multivariate* model when asset prices are cointegrated may be worthwhile because it reveals information about the long-run equilibrium in the system. If a spread is found to be mean reverting we know that, wherever one series is in several years' time, the other series will be right along there with it. Cointegrated asset prices have a common stochastic trend; they are 'tied together' in the long run because spreads are mean reverting, even though they might drift apart in the short run. In fact, cointegration can be thought of as a generalization of many 'common trends' methodologies such as principal component analysis.

The connection between these two methodologies is that a principal component analysis of cointegrated variables will yield the common stochastic trend as the first principal component (Gourieroux *et al.* 1991). But the outputs of the two analyses differ: principal components gives two or three series which can be used to approximate a much larger set of series (such as the yield curve); cointegration gives all possible stationary linear combinations of a set of random walks.

A slightly more technical description is necessary now: two integrated series are termed 'cointegrated' if there is a weighted sum (linear combination) of these series, called the 'cointegrating vector' and denoted  $z$ , that is stationary. In mathematical terms,  $x$  and  $y$  are cointegrated if  $x, y \sim I(1)$  but there exists  $\alpha$  such that  $z = x - \alpha y \sim I(0)$ . The definition of cointegration given in Engle & Granger (1987) is far more general than this, but a simple definition presented here is sufficient for the purposes of this paper.

When only two integrated series are considered for cointegration, there can be at most one cointegrating vector, because if there were two cointegrating vectors the original series would have to be stationary. More generally, cointegration exists between  $n$  integrated series if there exists at least one, but no more than  $n - 1$ , linear combinations that are stationary. Each stationary linear combination is a cointegrating vector. They act like 'bonds' in the system, and so the more cointegrating vectors found, the more the coherence and co-movements in the prices. Yield curves have very high cointegration: if each of the  $n - 1$  independent spreads is mean reverting, there will be  $n - 1$  cointegrating vectors (see §3).

### (b) *Statistical techniques*

There is an enormous literature about the merits of different methods of finding cointegration, starting with the classic 'Engle–Granger' methodology explained in their 1987 paper and subsequently refined in Engle & Yoo (1987). In the Engle–Granger method one simply performs a 'cointegrating' regression between the integrated series, and then tests the residual for stationarity. A number of different stationarity tests are relevant (see Choi 1992; Cochrane 1991; Dickey & Fuller 1979; Schmidt & Phillips 1992; Phillips & Perron 1988; Wang & Yau 1994), the most popular in this context being the Augmented Dickey Fuller (ADF) test (see Alexander 1994a; Alexander & Thillainathan 1996). Note that there is often a case to include a time trend in the regression or to omit the constant: more details may be found in Mackinnon (1994) and in standard econometrics texts.

In the case of only two log prices  $x$  and  $y$ , the Engle–Granger method is to perform an ordinary least squares regression

$$x_t = c + \alpha y_t + \varepsilon_t$$

and then test the residuals for stationarity. Note that it is only valid to regress log prices on log prices when these log prices are cointegrated. Only then will the residuals be stationary. Of course, the same remarks apply to the actual price data, but it is more usual to employ log prices since then residuals are returns and coefficients are allocation weights.

If  $\varepsilon$  is stationary, then  $x$  and  $y$  are cointegrated and the cointegrating vector is  $x - \alpha y$ . In this case it does not matter which of  $x$  or  $y$  is taken as the dependent variable. But when there are more than two series, the Engle–Granger method may suffer from bias and it should be absolutely clear which series is to be used as the dependent variable in the regression.

Johansen’s methodology for investigating cointegration in a multivariate system is commonly regarded as superior to the Engle–Granger method, particularly when the number of variables is greater than two (Johansen 1988; Johansen & Juselius 1990). The Johansen tests are based on the eigenvalues of a stochastic matrix and in fact reduce to a canonical correlation problem similar to that of principal components. They have no bias and their power function has better properties: the Johansen tests seek the linear combination which is most stationary whereas the Engle–Granger tests, being based on ordinary least squares, seek the linear combination having minimum variance.

However, for many financial applications of cointegration there are good reasons for choosing Engle–Granger as the preferred methodology. First, it is very straightforward to implement; secondly, in risk-management applications it is generally the Engle–Granger criterion of minimum variance, rather than the Johansen criterion of maximum stationarity, which is paramount; thirdly, there is often a natural choice of dependent variable in the cointegrating regressions (for example, in equity index arbitrage); and finally, the Engle–Granger small-sample bias is not necessarily going to be a problem anyway since sample sizes are generally quite large in financial analysis and the cointegrating vector is super consistent.

### (c) *Causality and error correction*

The mechanism which ties cointegrated series together is a ‘causality’, not in the sense that if we make a structural change to one series the other will change too, but in the sense that turning points on one series precede turning points in the other (see Granger 1988; Covey & Bessler 1992). The strength and directions of this ‘Granger causality’ can change over time, there can be bi-directional causality, or the direction of causality can change.

The causal flows that must exist in any cointegrated system are revealed during the second stage of cointegration modelling, namely the building of an ‘error correction model’ (ECM). This is a dynamic model based on correlations of returns but with the constraint that short-run deviations from the long-run equilibrium will eventually be corrected. In the simplest case that there are two cointegrated log price series  $x$  and

$y$ , the ECM takes the form

$$\begin{aligned}\Delta x_t &= \alpha_1 + \sum_{i=1}^{m_1} \beta_{1i} \Delta x_{t-i} + \sum_{i=1}^{m_2} \beta_{2i} \Delta y_{t-i} + \gamma_1 z_{t-1} + \varepsilon_{1t}, \\ \Delta y_t &= \alpha_2 + \sum_{i=1}^{m_3} \beta_{3i} \Delta x_{t-i} + \sum_{i=1}^{m_4} \beta_{4i} \Delta y_{t-i} + \gamma_2 z_{t-1} + \varepsilon_{2t},\end{aligned}$$

where  $\Delta$  denotes the first-difference operator,  $z$  is the cointegrating vector  $x - \alpha y$ , and the lags lengths and coefficients are determined by ordinary least squares regression. The generalization to more than two variables is straightforward, with one equation for each variable in the system and each equation having up to  $n - 1$  ‘disequilibrium terms’ corresponding to the cointegrating vectors. More details of short-run dynamics in cointegrated systems may be found in Baillie & Bollerslev (1994), Proietti (1997) and in any of the texts already cited.

The reason this model is called an ‘error-correction’ mechanism is that it will be found empirically that  $\gamma_1 < 0$  and  $\gamma_2 > 0$ . The final ‘disequilibrium term’ in the model will therefore constrain deviations from the long-run equilibrium so that errors are corrected. If  $z$  is large and positive, this will have a negative effect on  $\Delta x$  because  $\gamma_1 < 0$  and  $x$  will decrease, whereas the effect on  $\Delta y$  is positive because  $\gamma_2 > 0$  and so  $y$  will increase. Both have the effect of reducing  $z$ , and in this way errors are corrected.

When  $x$  and  $y$  are cointegrated log asset prices, the error-correction model will capture dynamic correlations and causalities between their returns. If the coefficients on the lagged  $y$  returns in the  $x$  equation are found to be significant, then turning points in  $y$  will lead turning points in  $x$ . That is,  $y$  ‘Granger causes’  $x$ . There must be causalities when a spread is mean reverting and two asset prices are moving in line, but the direction of causality may change over time. For example, in the ‘price-discovery’ relationship between spot and futures it is often found that futures prices lead the spot, but at some times an ECM analysis shows that spot may very well lead futures prices (see § 3).

### 3. A review of practical applications to cointegrated financial assets

It is only recently that market practitioners have found important applications of the vast body of academic research into cointegration in financial markets. In this respect, financial analysts have been characteristically slow in adopting a new modelling approach. But, at least during the last few years, asset management companies have been investing in quantitative research projects to use cointegration for buy-side strategies, hedge funds are employing the long-short strategies indicated by cointegrating vectors, and quantitative methods for pricing spread options are being developed. This section reviews some of the publications that are relevant for the implementation of cointegration modelling of financial markets and at least hints at the potential for practical application.

#### (a) Market efficiency

Any system of financial asset prices with a mean-reverting spread will have some degree of cointegration, even though the Granger causalities inherent in such a system

contradict the efficiency of financial markets (Dwyer & Wallace 1992). For example, two log exchange rates are unlikely to be cointegrated since their difference is the cross rate and if markets are efficient that rate will be non-stationary. There is, however, some empirical evidence of cointegration between three or more exchange rates (see Goodhart 1988; Hakkio & Rush 1989; Coleman 1990; Alexander & Johnson 1992, 1994; Alexander 1995; MacDonald & Taylor 1994; Nieuwland *et al.* 1994).

Many financial journals, for example, the Journal of Futures Markets, contain papers on cointegration between spot and futures prices. Since spot and futures are tied together, the basis is the mean-reverting cointegrating vector, and the associated ECM has become the focus of research into the price-discovery relationship. This has been found to change considerably over time (see MacDonald & Taylor 1988; Nugent 1990; Bessler & Covey 1991; Bopp & Sitzer 1987; Chowdray 1991; Lai & Lai 1991; Khoury & Yourougou 1991; Schroeder & Goodwin 1991; Schwartz & Laatsch 1991; Beck 1994; Lee 1994; Schwartz & Szakmary 1994; Brenner & Kroner 1995; Harris *et al.* 1995; Alexander 1999). There is considerable scope for futures traders to develop models that exploit this relationship, as has been the case in some of the London Houses.

ECMs can be used to detect possibilities for spot-futures arbitrage, but the finding of a statistically significant cointegration–causality relationship is not necessarily sufficient for trading and hedging. The only way of telling this is to conduct a thorough profit and loss analysis based on model training and out-of-sample testing, and an example of such a program is outlined in § 4.

#### (b) *Related commodity markets*

Commodity products that are based on the same underlying, such as soya bean crush and soya bean oil, should be cointegrated if carry costs are mean reverting. However, the evidence for this seems rather weak, and the academic argument that related commodities such as different types of metals should be cointegrated is even more difficult to justify empirically. Brenner & Kroner (1995) present a useful survey of the literature in this area and conclude that the idiosyncratic behaviour of carry costs makes it very difficult to use ECMs in commodity markets. High-frequency technical traders dominate these markets and it is unlikely that cointegration between related spot markets is robust enough for trading. Nevertheless, cointegration across related commodity types can be quite strong—for example, the long-term relationship between crude oil, heating oil, natural gas and the ‘crack’ spread (from ‘cracking’, which is the process of extracting petroleum from crude oil).

#### (c) *Option pricing with cointegration*

Modelling cointegrated assets in Brownian diffusion processes is one of the many interesting problems presented at the Finance Seminars in the Newton Institute at Cambridge University during the summer of 1996. Jin-Chuan Duan and Stan Pliska took up this challenge and have recently produced an excellent study of the theory of option valuation with cointegrated asset prices (Duan & Pliska 1999). Their discrete time model for valuing spread options has the continuous limit of a system driven by four correlated Brownian motions, one for each asset (in which the ECM is incorporated) and one for each stochastic volatility. The stochastic processes for the asset returns have an additional disequilibrium term to ensure that deviations from



equilibrium are corrected, so stationary spreads are imposed. Their Monte Carlo results show that cointegration may have a substantial influence on option prices when volatilities are stochastic. But when volatilities are constant the model simplifies to one of simple bivariate Brownian motion and the standard Black–Scholes results are recovered.

(d) *Cointegrated asset markets*

It is not surprising that fixed-income markets are easily modelled with cointegration. Bond yields are random walks that are most probably cointegrated across different maturities within a given country. Wherever the 1 month yield is in 10 or 20 years time, the 3 month yield will be right along there with it, because the spread has finite variance. More generally, in a yield curve of  $n$  maturities, each of the  $n - 1$  independent spreads is a cointegrating vector, assuming it is mean reverting.

Cointegration and correlation go together in the yield curve, and we often find strongest cointegration at the short end, where correlations are highest (see Bradley & Lumpkin 1992; Hall *et al.* 1992; Alexander & Johnson 1992, 1994; Davidson *et al.* 1994; DeGennaro *et al.* 1994; Lee 1994; Boothe & Tse 1995; Brenner *et al.* 1996). ECMs of very short rates sometimes reveal surprising results about the directions of causal flows between different maturities, and it is not always the very short rates that lead the system.

There is some evidence of cointegration in international bond markets and in international equity markets, but arbitrage possibilities seem quite limited (Clare *et al.* 1995; Corhay *et al.* 1993; Karfakis & Moschos 1990; Kasa 1992; Smith *et al.* 1993). However, in recent years the US market does appear to be somewhat of a leader in international equity and bond markets (Alexander 1994*b*; Masih 1997).

Cerchi & Havenner (1988) and Pindyck & Rothenberg (1992) investigate stock price cointegration. Since an equity index is by definition a weighted sum of the constituents, there should be some sufficiently large basket, which is cointegrated with the index, if the index weights do not change too much over time. Different stock market indices within a given country should also be cointegrated when industrial sectors maintain relatively stable proportions in the economy, and market indices of two different countries should be cointegrated if purchasing power parity holds. These ideas are currently being developed by a number of asset management companies and the next section of this paper introduces one such cointegration based equity index tracking and hedging model.

#### 4. An empirical model for cointegrated international equity markets

As already mentioned in the introduction, risk-return analysis alone has nothing to ensure that tracking errors are stationary. In standard models, tracking errors may quite possibly be random walks, so the replicating basket can drift arbitrarily far from the benchmark unless it is frequently rebalanced. Portfolio replication strategies that can guarantee mean-reverting tracking errors must have cointegration as the basis, however obscured. This section presents a method for modelling cointegration that has been developed into a powerful asset management tool for hedging and index arbitrage.†

† Details available from Pennoyer Capital Management, Inc., 230 Park Avenue, Suite 1000, New York, NY 10169, USA, or from [www.pennoyer.net](http://www.pennoyer.net).

The World Integrated Equity Selector (WIES) is a quantitative tool for international equity investment that is based on cointegration between the European, Asian and Far East (EAFE) Morgan Stanley index and the constituent country indices. In the first stage, countries are selected according to preferences of investors, which may be quite specific. For example, 50% of the fund may be allocated to the UK, or the manager may wish to avoid Japan entirely. The model may be run with or without short sales, but since the constraint of no short sales will restrict possibilities of cointegrating baskets it has been applied primarily for hedge funds.

In a simple example a fund manager may wish to go long–short in, say, eight different countries, with the EAFE index as benchmark. The problem then becomes one of selecting the basket of eight countries that are currently most highly cointegrated with the EAFE index. In another example, small asset management companies might seek a benchmark return of, say, 2% per annum above the EAFE index. A further alternative is to place a bound on the tracking-error variance, so the number of non-zero allocations is not specified. This last preference, which is most often used, may result in selecting as few as four or as many as 20 different countries, depending on the time and the bound set for the tracking-error variance. But whatever the preferences of the investor, there are going to be a huge number of country combinations to test since the EAFE index currently contains 20 country indices.

The selection criterion used is a highly technical proprietary part of the model. However, once the countries have been targeted the methodology is easily explained. Common-trends analysis should employ data covering entire business cycles, so possibly 10 or more years of data should be considered.

The training-period length is one of the model parameters that are optimized by testing the model with post-sample performance measures such as information ratios and correlations. More details of this training and testing procedure are now given. In a given month, suppose we have chosen to employ long–short strategies between  $n$  different countries, whose log price indices in local currencies<sup>†</sup> are  $x_1, \dots, x_n$ . We perform a cointegration regression of the log EAFE price index  $y$  on these:

$$y_t = \alpha_0 + \alpha_1 x_1 + \dots + \alpha_n x_n + \varepsilon_t.$$

Potential allocations are obtained, and the tracking error is the cointegrating vector (being the residual from the cointegrating regression). The technique of constrained ordinary least squares is applied for specific preferences. For example, the no short sales preference is obtained by constraining coefficients to be positive, or for a specific weight in a country the value of the coefficient is fixed before the regression. The more stationary the tracking error, the greater the cointegration between the EAFE index and the candidate portfolio.

The strength of this hedging or tracking methodology lies in its long-term performance; therefore, the portfolios should be viewed as long-term investments. Only those portfolios showing realistic turnover projections are considered. The common trends on which this model is based are very long term and so for most sensible preferences the instability of country allocations is not a problem.

Low turnover goes hand in hand with autocorrelated and cross-correlated information ratios. In-sample information ratios are always zero, because the residuals from

<sup>†</sup> A similar method may be applied to domestic currency indices, but the foreign exchange risk and return is better dealt with separately. Using local currencies ensures that all returns are attributable to equity performance, and currency basket options may be used to hedge the foreign exchange risk.

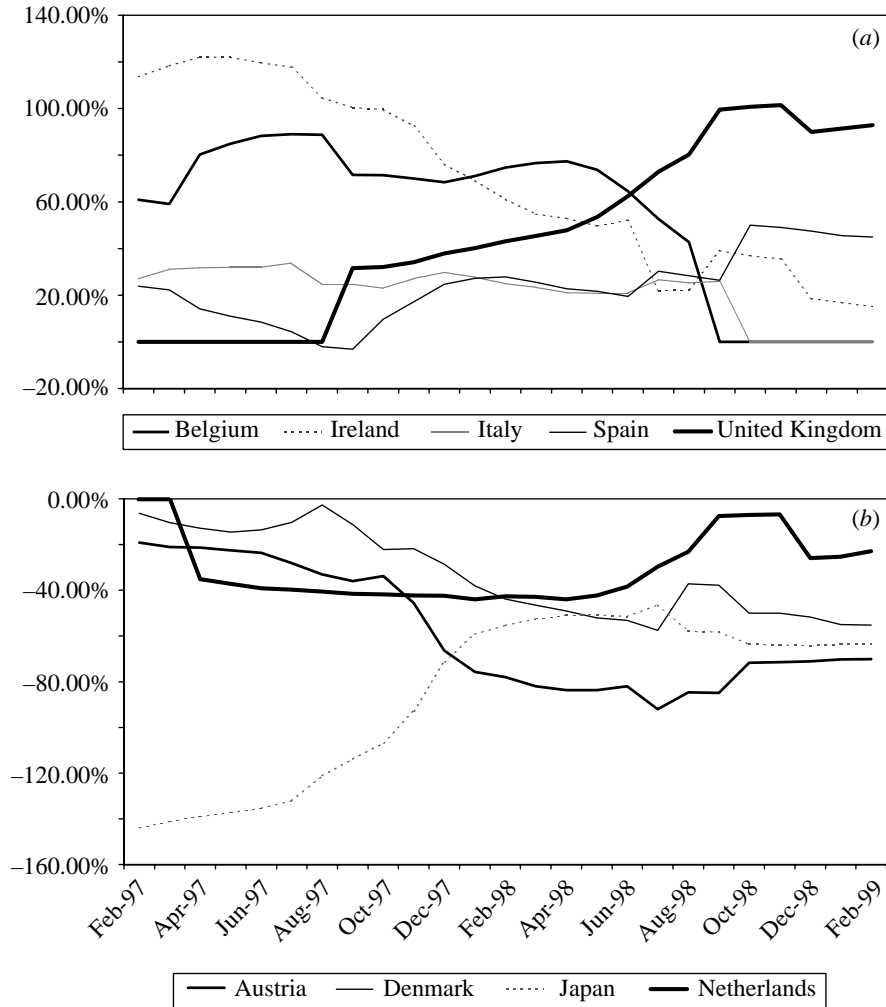


Figure 2. WEIS Hedge fund (a) long and (b) short allocations.

ordinary least squares regression have zero mean; however, the post-sample information ratio is a very important performance measure, as is the cross-correlation between one-month and one-year information ratios, and the autocorrelation of information ratios. If one-month information ratios are high, and they are highly correlated with one-year information ratios, then we know that portfolios selected on the basis of monthly testing are likely to remain good for much longer. Moreover, if information ratios are consistently high and autocorrelated, the preference criteria are stable over time. Highly autocorrelated information ratios show that if last month's selection performed well, then next month's selection is also likely to be good.

In summary, the preferences to be set include: a choice of fixed countries or a variable number of countries; constraints on country allocations that can be very flexible, such as at least 50% in the UK, or no short sales, or 0% in Japan; the length

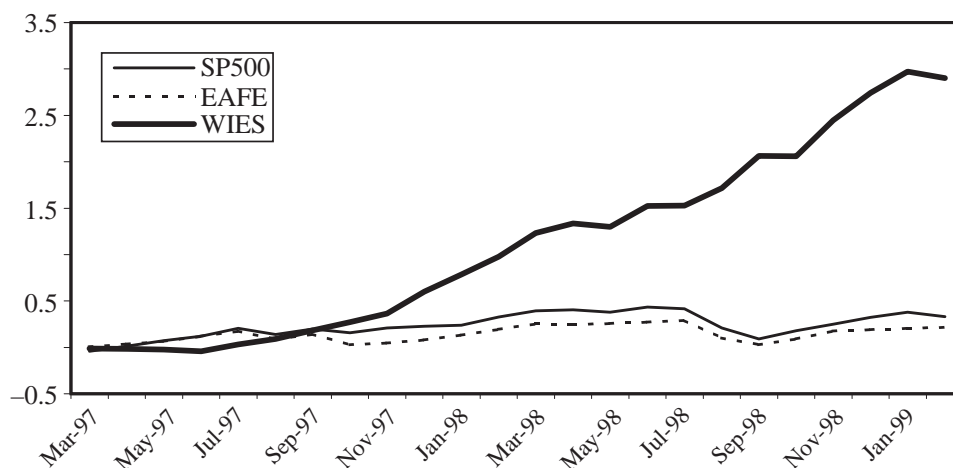


Figure 3. Cumulative returns from WEIS hedge fund.

of look-back period for the model training; and any 'plus' over the EAFE if it is used as a benchmark. The diagnostics analysed include: in-sample stationarity tests for the tracking error (ADFs); monthly turnovers; and information ratios of post-sample performance over a testing period that is usually 12 months.

Some examples of running the model in practice are now presented. Two different types of preferences are illustrated, one being a hedge portfolio and the other being an index-tracking portfolio that is constrained to allow no short sales and nothing to be invested in Japan. They are presented purely as examples of the model back-testing and validation methodology and the illustrative portfolios are not intended as recommendations for practical use.

The output in table 1 is for outperforming the EAFE index plus 2% per annum, with no short sales and no investment in Japan. Concentration/diversification preferences have been set for investment in up to nine countries, and with a four-year training period. The training output is given in the first three columns and the information ratio (IR) for a post-sample testing period are reported to the right of the table. Each row in the table corresponds to one portfolio that has been chosen to be most highly cointegrated with the benchmark and also constrained to reflect the preferences set by the investor (in this case, no short sales and none of the fund in Japan). The portfolios are rebalanced monthly or quarterly. The actual country allocations are not shown, but see table 2 for country allocations when WIES is used for the hedge fund.

A turnover index is first calculated as the sum of the absolute change in weights. This generally lies between 0 and 2.0, the latter occurring when there is 100% turnover in all assets (i.e. the entire portfolio moves out of one set of assets, then into another). The turnover per cent reported in table 1 is simply this turnover index divided by two and expressed as a percentage. Turnovers are generally low, except between May 1996 and May 1997. But since the model is being constrained (much more constrained than for the hedging example given below), the turnovers are expected to be more variable. The ADFs show that in-sample tracking errors are stationary most of the time, but they are not always significant. One may conclude

Table 1. *Back-testing the preferences of no short sales and 0% in Japan*

(Notes: ADF is the in-sample ADF statistic on tracking-error stationary; IR is the post-sample information ratio, which is the mean-tracking error/s.d. tracking error over the testing period, relative to the EAFE index; other diagnostics include autocorrelation of information ratios (0.7377) and cross-correlation between one-month and one-year information ratios (0.4902).)

in-sample training period is four years to:	ADF	turnover (%)	post-sample testing period is one year to:	IR
31 December 1993	-2.4159	0	31 December 1994	-0.3954
31 January 1994	-2.4286	13.7	31 January 1995	-0.3416
28 February 1994	-2.7067	5.5	28 February 1995	0.6716
31 March 1994	-2.4454	6.3	31 March 1995	-0.0170
30 April 1994	-2.2907	7.25	30 April 1995	-0.2239
31 May 1994	-2.7270	3.7	31 May 1995	1.4278
30 June 1994	-2.7474	0.5	30 June 1995	1.9063
31 July 1994	-2.6268	0.95	31 July 1995	1.1598
31 August 1994	-2.5351	1.25	31 August 1995	0.8325
30 September 1994	-2.5852	1.9	30 September 1995	1.7035
31 October 1994	-2.5662	1.05	31 October 1995	2.2529
30 November 1994	-2.4222	1.8	30 November 1995	1.9891
31 December 1994	-2.5741	3.95	31 December 1995	2.9024
31 January 1995	-2.4951	7.1	31 January 1996	2.2928
28 February 1995	-2.1847	5.75	28 February 1996	1.7201
31 March 1995	-2.2816	6.5	31 March 1996	1.9084
30 April 1995	-2.1831	14.9	30 April 1996	1.1432
31 May 1995	-1.8767	8.4	31 May 1996	-0.2004
30 June 1995	-1.6848	9.6	30 June 1996	-1.1051
31 July 1995	-2.2644	5.25	31 July 1996	-0.8722
31 August 1995	-2.4214	3.85	31 August 1996	0.6893
30 September 1995	-3.1198	5.45	30 September 1996	1.0470
31 October 1995	-3.2954	11.3	31 October 1996	1.6794
30 November 1995	-3.3002	13.4	30 November 1996	1.3118
31 December 1995	-3.4768	14.35	31 December 1996	1.2892
31 January 1996	-3.1175	1.65	31 January 1997	0.6507
28 February 1996	-3.0682	1	28 February 1997	0.7446
31 March 1996	-3.4111	5.55	31 March 1997	1.4376
30 April 1996	-3.1976	4.45	30 April 1997	1.2103
31 May 1996	-3.0054	13.55	31 May 1997	1.1804
30 June 1996	-3.3631	32.8	30 June 1997	1.4211
31 July 1996	-3.8745	19.5	31 July 1997	1.4205
31 August 1996	-3.4884	15.7	31 August 1997	1.4748
30 September 1996	-3.4569	3.45	30 September 1997	1.9127
31 October 1996	-3.1545	3.4	31 October 1997	2.0667
30 November 1996	-3.0922	2.95	30 November 1997	2.3661
31 December 1996	-2.4080	22.15	31 December 1997	2.3083
31 January 1997	-3.7039	16.8	31 January 1998	1.6091
28 February 1997	-1.8348	46.55	28 February 1998	1.2108

Table 1. *Cont.*

in-sample training period is four years to:	ADF	turnover (%)	post-sample testing period is one year to:	IR
31 March 1997	-0.0642	72.65	31 March 1998	1.4225
30 April 1997	-0.9878	8.3	30 April 1998	1.5957
31 May 1997	-2.8355	82.25	31 May 1998	0.7172
30 June 1997	-2.7019	1.45	30 June 1998	0.7305
31 July 1997	-2.9622	1.2	31 July 1998	0.8534
31 August 1997	-3.0783	1.75	31 August 1998	1.2016
30 September 1997	-3.0405	2.45	31 August 1998	1.2284
31 October 1997	-3.0546	1.6	31 August 1998	1.5020
30 November 1997	-3.1603	1.3	31 August 1998	1.4473
31 December 1997	-2.9487	2	31 August 1998	0.5557
31 January 1998	-3.1410	1.45	31 August 1998	1.0104
28 February 1998	-3.1720	0.1	31 August 1998	1.8855
31 March 1998	-3.1131	16.95	31 August 1998	1.5793
30 April 1998	-3.1963	2.2	31 August 1998	0.7399
31 May 1998	-3.3073	2.15	31 August 1998	1.0287
30 June 1998	-3.5255	13.2	31 August 1998	1.6629
31 July 1998	-3.7153	12.55	31 August 1998	1.2587
31 August 1998	-3.9901	7.4		

that the portfolios are weakly cointegrated with the EAFE plus 2%. What is encouraging is that cointegration is more significant towards the end of the sample, since all ADFs exceed  $-3$  during 1998, and this is what matters when choosing the WIES parameters for current allocations.

The post-sample information ratios are almost all positive. Looking at the IRs during 1998 one might reasonably expect a value of around 1 for a portfolio chosen on the basis of these preferences. It is possible to be reasonably confident of this because the IRs are highly autocorrelated (the autocorrelation coefficient is 0.7377). There is also a significant positive cross-correlation between the one-month and the 12-month information ratios of 0.4902, showing that if a portfolio starts well during the first month it is likely to stay good.

This combination of high autocorrelation and high cross-correlation in the information ratios is a great advantage, since it enables one-month ahead decisions to be made. The information ratios at the end of the training and testing output refer only to a few months of post-sample testing. Therefore, one must seek a combination of high-current IRs, high autocorrelation, and high cross-correlation in IRs to predict that next month a portfolio chosen on the basis of these preferences will outperform the EAFE index.

The example of tracking EAFE plus 2%, no short sales and 0% in Japan has produced some promising results, but many other preferences would not. For example, if we were to stipulate that 50% of the fund be allocated to the UK, it would be difficult to find portfolios that are highly cointegrated with the EAFE index.

Tables 2 and 3 report results from using the model for a hedge. When the model is used in this way the problem becomes one of finding a long portfolio and a short

Table 2. *WIES hedge fund allocations*

(Notes. The coefficients are not normalized. For example, in the last month they sum to 2.3, which implies a total leverage of 4.6 to 1.)

date	Australia	Austria	Belgium	Denmark	Finland	France
February 1997	0.00	-19.10	60.86	-6.32	-12.45	0.00
March 1997	0.00	-21.10	59.10	-10.39	-14.49	0.00
April 1997	0.00	-21.29	80.24	-12.79	-14.54	0.00
May 1997	0.00	-22.50	84.87	-14.54	-14.59	0.00
June 1997	0.00	-23.65	88.26	-13.57	-13.49	0.00
July 1997	0.00	-28.05	89.04	-10.28	-12.64	0.00
August 1997	0.00	-33.08	88.78	-2.64	0.00	0.00
September 1997	0.00	-35.91	71.51	-11.26	0.00	0.00
October 1997	0.00	-33.75	71.50	-22.13	0.00	0.00
November 1997	0.00	-45.59	69.97	-21.76	0.00	0.00
December 1997	0.00	-66.32	68.43	-28.56	0.00	0.00
January 1998	0.00	-75.64	71.13	-37.98	0.00	0.00
February 1998	0.00	-78.04	74.68	-43.87	0.00	0.00
March 1998	0.00	-81.95	76.69	-46.50	0.00	0.00
April 1998	0.00	-83.65	77.42	-49.09	0.00	0.00
May 1998	0.00	-83.68	73.72	-52.12	0.00	0.00
June 1998	0.00	-82.05	64.57	-53.22	0.00	0.00
July 1998	0.00	-92.09	52.66	-57.48	0.00	3.32
August 1998	0.00	-84.61	42.73	-37.16	0.00	0.00
September 1998	0.00	-84.88	0.00	-37.78	0.00	0.00
October 1998	0.00	-71.67	0.00	-50.08	0.00	0.00
November 1998	0.00	-71.47	0.00	-49.98	0.00	0.00
December 1998	0.00	-71.04	0.00	-51.72	0.00	0.00
January 1999	0.00	-70.22	0.00	-54.97	0.00	0.00
February 1999	0.00	-70.11	0.00	-55.28	0.00	0.00

portfolio that are highly cointegrated. Figure 2*a, b* illustrates the major long and short positions that would have obtained from using these preferences for the WIES hedge portfolio. Belgium, Ireland, Italy, Spain and the UK were all significantly long during this period, and Austria, Denmark, Japan and The Netherlands were short positions.

Table 3 shows monthly and cumulative returns that would have obtained from hedging with these preferences since March 1997, compared with those from tracking EAFE and Standard & Poor (SP500). These are some of the relevant results used during back-testing current preferences while training for the March 1999 allocations. Note that the returns that are reported during in-sample training are not the same as the realized returns from the managed portfolio. This is because the back-testing results are for just one preference set. In practice, we are changing the optimal preferences each month, as different alphas, different training periods and different constraints on allocations appear more or less profitable during back-testing.

Table 2. *Cont.*

date	Germany	Hong Kong	Ireland	Italy	Japan	Malaysia	The Netherlands
February 1997	0.00	0.00	113.59	27.01	-143.97	0.00	0.00
March 1997	0.00	0.00	118.27	31.09	-141.07	0.00	0.00
April 1997	0.00	0.00	122.05	31.76	-139.10	0.00	-35.17
May 1997	0.00	0.00	121.96	31.97	-137.17	0.00	-37.24
June 1997	0.00	0.00	119.64	31.94	-135.43	0.00	-39.13
July 1997	0.00	0.00	117.69	33.83	-132.15	0.00	-39.69
August 1997	10.07	0.00	104.63	24.58	-121.10	0.00	-40.58
September 1997	10.42	0.00	100.43	24.68	-113.91	0.00	-41.53
October 1997	3.29	0.00	99.64	23.04	-106.79	0.00	-41.83
November 1997	-3.44	0.00	92.50	27.08	-92.92	0.00	-42.27
December 1997	1.79	0.00	76.14	29.76	-71.34	0.00	-42.38
January 1998	7.56	0.00	69.18	27.78	-59.27	0.00	-43.94
February 1998	11.71	0.00	61.11	24.88	-55.40	0.00	-42.67
March 1998	18.06	0.00	54.75	23.35	-52.51	0.00	-42.87
April 1998	22.51	0.00	52.96	21.08	-50.95	0.00	-43.98
May 1998	24.33	0.00	49.61	20.85	-50.73	0.00	-42.23
June 1998	21.55	0.00	52.16	20.95	-51.44	0.00	-38.42
July 1998	29.53	0.00	21.62	26.67	-46.35	0.00	-29.62
August 1998	18.65	0.00	22.45	25.24	-58.03	8.55	-23.16
September 1998	19.52	0.00	39.05	25.85	-58.24	7.79	-7.51
October 1998	18.48	0.00	36.94	0.00	-63.55	0.00	-7.05
November 1998	18.67	0.00	35.51	0.00	-63.98	0.00	-6.76
December 1998	46.26	0.00	18.47	0.00	-64.16	0.00	-25.90
January 1999	47.00	0.00	16.83	0.00	-63.72	0.00	-25.28
February 1999	46.20	0.00	15.09	0.00	-63.67	0.00	-22.88

## 5. Concluding remarks

The extensive literature on cointegrated asset prices concerns both market efficiencies (e.g. spot and futures prices) and market inefficiencies (e.g. international equity markets). The innovation of cointegration is the direct application of non-stationary data, whereas most other methods employ stationary data from the outset. Cointegration arises naturally between a number of asset markets, and when cointegration is not taken into account there is a danger of mis-pricing, over-hedging and increased transaction costs.

Cointegration, perhaps more than any other time-series methodology, has been used pervasively and skillfully in econometrics, but there is a danger of forming false conclusions. There is sometimes a tendency for time-series modelling to be led by the technique, rather than any real relationship between the random variables in a multivariate system. But if the finding of a relationship between underlyings is not merely an artefact of the statistical methods used, and is really a result of



Table 2. *Cont.*

date	New Zealand	Norway	Singapore	Spain	Sweden	Switzerland	UK
February 1997	11.32	-25.49	0.00	23.81	-24.13	-5.14	0.00
March 1997	14.53	-26.15	0.00	22.22	-26.84	-5.18	0.00
April 1997	0.00	-26.71	0.00	14.12	-17.78	19.21	0.00
May 1997	0.00	-25.64	0.00	10.97	-16.31	18.25	0.00
June 1997	0.00	-25.83	0.00	8.44	-14.89	17.72	0.00
July 1997	0.00	-25.26	0.00	4.30	-14.75	17.97	0.00
August 1997	0.00	-32.37	0.00	-2.02	-14.81	18.54	0.00
September 1997	0.00	-30.87	0.00	-3.22	-3.52	1.53	31.65
October 1997	0.00	-32.73	0.00	9.62	-3.35	1.45	32.03
November 1997	0.00	-32.76	0.00	17.20	-3.14	1.04	34.09
December 1997	0.00	-27.32	0.00	24.75	-1.52	-1.33	37.89
January 1998	0.00	-23.41	0.00	27.26	1.72	-4.53	40.14
February 1998	0.00	-18.91	0.00	27.81	6.03	-10.43	43.10
March 1998	0.00	-15.02	0.00	25.54	8.80	-13.73	45.40
April 1998	0.00	-11.93	0.00	22.81	10.71	-15.67	47.79
May 1998	0.00	-8.30	0.00	21.63	12.08	-18.64	53.49
June 1998	0.00	-5.81	0.00	19.53	12.18	-22.54	62.52
July 1998	0.00	6.16	0.00	30.24	11.03	-28.55	72.87
August 1998	0.00	-1.02	0.00	28.38	9.20	-31.50	80.27
September 1998	0.00	1.30	0.00	26.45	-5.03	-26.06	99.54
October 1998	0.00	1.38	15.42	50.01	-4.21	-26.45	100.77
November 1998	0.00	2.53	15.23	49.00	-3.58	-26.69	101.51
December 1998	0.00	5.13	14.24	47.47	8.16	-16.87	89.96
January 1999	0.00	9.74	12.08	45.52	8.77	-17.14	91.40
February 1999	0.00	10.82	11.63	44.88	8.89	-18.51	92.94

market forces, then it should be corroborated by alternative statistical methods. It will 'shine through' a variety of statistical investigations, using a number of different methodologies. However, if supposedly profitable relationships are only revealed when a finely tuned device such as cointegration is applied, then there is a good chance that the relationships are not sufficiently robust to be workable in practice.

Cointegration provides a rigorous scheme that may be implemented with basic statistical techniques. Maximum likelihood or numerical optimization methods are not necessary for the modeller to gain a crude insight to the common-trends and error-correction dynamics of a cointegrated system. And if no strong evidence is presented within a superficial investigation, it is unlikely that the trading P&Ls would support the use of the model.

This paper has focused on the use of cointegration for equity-index tracking and hedging of international equity portfolios. From the above remarks it should be clear that, however rigorous the modelling approach, it is the training and testing results on candidate portfolios that are paramount. It has been shown how such validation

Table 3. WIES hedge fund returns

date	monthly return			cumulative return		
	SP500	EAFE	WIES	SP500	EAFE	WIES
March 1997	-4.26	0.48	-1.11	-4.26	0.48	-1.11
April 1997	5.84	2.94	-0.43	1.33	3.44	-1.53
May 1997	5.86	2.89	-0.79	7.27	6.43	-2.31
June 1997	4.35	5.28	-2.00	11.94	12.05	-4.27
July 1997	7.81	5.07	7.72	20.68	17.72	3.12
August 1997	-5.74	-7.56	5.90	13.75	8.82	9.20
September 1997	5.32	5.00	8.20	19.80	14.26	18.16
October 1997	-3.45	-9.89	7.65	15.67	2.96	27.20
November 1997	4.46	1.74	7.32	20.83	4.76	36.52
December 1997	1.57	3.14	17.18	22.72	8.05	59.96
January 1998	1.02	4.66	11.50	23.98	13.08	78.36
February 1998	7.04	5.55	10.83	32.70	19.35	97.67
March 1998	4.99	4.99	12.96	39.33	25.30	123.29
April 1998	0.91	-0.69	4.55	40.59	24.44	133.46
May 1998	-1.88	0.94	-1.55	37.95	25.60	129.84
June 1998	3.94	1.13	9.81	43.39	27.02	152.39
July 1998	-1.16	1.57	0.16	41.72	29.01	152.80
August 1998	-14.58	-14.48	7.39	21.06	10.34	171.49
September 1998	-9.90	-6.82	12.82	9.08	2.81	206.28
October 1998	8.10	6.15	-0.17	17.91	9.14	205.77
November 1998	6.10	7.63	12.73	25.10	17.47	244.71
December 1998	5.80	1.24	8.61	32.36	18.93	274.38
January 1999	4.20	1.27	6.12	37.92	20.43	297.31
February 1999	-3.43	0.91	-1.82	33.19	21.53	290.09

procedures may be put in place, and results have been presented for both hedging and index tracking.

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## References

- Alexander, C. 1994a History debunked. *RISK* **7**, 59–63.
- Alexander, C. 1994b Cofeatures in international bond and equity markets. In *University of Sussex Discussion Papers in Economics*, no. 94/1.
- Alexander, C. 1995 Common volatility in the foreign exchange market. In *Applied Financial Economics*, **5**, 1–10.
- Alexander, C. 1999 Correlation and cointegration in energy markets. In *Managing energy price risk*, 2nd edn. RISK Publications.
- Alexander, C. & Johnson, A. 1992 Are foreign exchange markets really efficient? *Econ. Lett.* **40**, 449–453.

*Phil. Trans. R. Soc. Lond. A* (1999)

- Alexander, C. & Johnson, A. 1994 Dynamic links. *RISK* **7**, 56–61.
- Alexander, C. & Thillainathan, R. 1996 The Asian connections. *Emerging Markets Investor* **2**, 42–47.
- Baillie, R. T. & Bollerslev, T. 1994 Cointegration, fractional cointegration, and exchange rate dynamics. *J. Finance* **49**, 737–745.
- Beck, S. E. 1994 Cointegration and market efficiency in commodities futures markets. *Appl. Econ.* **26**, 249–257.
- Bessler, D. A. & Covey, T. 1991 Cointegration—some results on cattle prices. *J. Futures Markets* **11**, 461–474.
- Booth, G. & Tse, Y. 1995 Long memory in interest rate futures markets: a fractional cointegration analysis. *J. Futures Markets* **15**.
- Bopp, A. E. & Sitzer, S. 1987 Are petroleum prices good predictors of cash value? *J. Futures Markets* **7**, 705–719.
- Bradley, M. & Lumpkin, S. 1992 The Treasury yield curve as a cointegrated system. *J. Financial Quantitative Analysis* **27**, 449–463.
- Brenner, R. J. & Kroner, K. F. 1995 Arbitrage, cointegration, and testing the unbiasedness hypothesis in financial markets. *J. Financial Quantitative Analysis* **30**, 23–42.
- Brenner, R. J., Harjes, R. H. & Kroner, K. F. 1996 Another look at alternative models of the short term interest rate. *J. Financial Quantitative Analysis* **31**, 85–108.
- Campbell, J. Y., Lo, A. W. & MacKinlay, A. C. 1997 *The econometrics of financial markets*. Princeton University Press.
- Cerchi, M. & Havenner, A. 1988 Cointegration and stock prices. *J. Econ. Dynamics Control* **12**, 333–346.
- Choi, I. 1992 Durbin–Hausmann tests for a unit root. *Oxford Bull. Econ. Statist.* **54**, 289–304.
- Chowdhury, A. R. 1991 Futures market efficiency: evidence from cointegration tests. *J. Futures Markets* **11**, 577–589.
- Clare, A. D., Maras, M. & Thomas, S. H. 1995 The integration and efficiency of international bond markets. *J. Business Finance Accounting* **22**, 313–322.
- Cochrane, J. H. 1991 A critique of the application of unit root tests. *J. Econ. Dynamics Control* **15**, 275–284.
- Coleman, M. 1990 Cointegration-based tests of daily foreign exchange market efficiency. *Econ. Lett.* **32**, 53–59.
- Corhay, C. A., Tourani Rad & Urbain, J.-P. 1993 Common stochastic trends in European stock markets. *Econ. Lett.* **42**, 385–390.
- Covey, T. & Bessler, D. A. 1992 Testing for Granger’s full causality. *Rev. Econ. Statist.* **72**, 146–153.
- Davidson, J., Madonia, G. & Westaway, P. 1994 Modelling the UK gilt-edged market. *J. Appl. Econometrics* **9**, 231–253.
- DeGennaro, R. P., Kunkel, R. A. & Lee, J. 1994 Modeling international long-term interest rates. *Financial Rev.* **29**, 577–597.
- Dickey, D. A. & Fuller, W. A. 1979 Distribution of the estimates for autoregressive time series with a unit root. *J. Am. Statistical Assoc.* **74**, 427–429.
- Duan, J.-C. & Pliska, S. 1999 *Option valuation with cointegrated asset prices*. (<http://www.bm.ust.hk/~jcduan/index.html>.)
- Dwyer, G. P. & Wallace, M. S. 1992 Cointegration and market efficiency. *J. Int. Money Finance* **11**, 318–327.
- Engle, R. F. & Granger, C. W. J. 1987 Cointegration and error correction: representation, estimation, and testing. *Econometrica* **55**, 251–276.
- Engle, R. F. & Yoo, B. S. 1987 Forecasting and testing in cointegrated systems. *J. Econometrics* **35** 143–159.
- Phil. Trans. R. Soc. Lond. A* (1999)

- Goodhart, C. 1988 The foreign exchange market: a random walk with a dragging anchor. *Economica* **55**, 437–460.
- Gourieroux, C., Monfort, A. & Renault, E. 1991 A general framework for factor models. Institut National de la Statistique et des Etudes Economiques, paper no. 9107.
- Granger, C. W. J. 1988 Some recent developments on a concept of causality. *J. Econometrics* **39**, 199–211.
- Hakkio, C. S. & Rush, M. 1989 Market efficiency and cointegration: an application to the sterling and Deutschmark exchange markets. *J. Int. Money Finance* **8**, 75–88.
- Hall, A. D., Anderson, H. M. & Granger, C. W. J. 1992 A cointegration analysis of Treasury bill yields. *Rev. Econ. Statist.* 116–126.
- Hamilton, J. D. 1994 *Time-series analysis*. Princeton University Press.
- Harris, F., McInish, T. H., Shoesmith, G. L. & Wood, R. A. 1995 Cointegration, error correction, and price discovery on informationally linked security markets. *J. Financial Quantitative Analysis* **30**.
- Hendry, D. F. 1986 Econometric modelling with cointegrated variables: an overview. *Oxford Bull. Econ. Statist.* **48**, 201–212.
- Hendry, D. F. 1995 *Dynamic econometrics*. Oxford University Press.
- Johansen, S. 1988 Statistical analysis of cointegration vectors. *J. Econ. Dyn. Control* **12**, 231–254.
- Johansen, S. & Juselius, K. 1990 Maximum likelihood estimation and inference on cointegration—with applications to the demand for money. *Oxford Bull. Econ. Statist.* **52**, 169–210.
- Karfakis, C. J. & Moschos, D. M. 1990 Interest rate linkages within the European monetary system: a time series analysis. *J. Money Credit Banking* **22**, 388–394.
- Kasa, K. 1992 Common stochastic trends in international stock markets. *J. Monetary Econ.* **29**, 95–124.
- Khoury N. T. & Yourougou, P. 1991 The informational content of the basis: evidence from Canadian barley, oats and canola futures markets. *J. Futures Markets* **11**, 69–80.
- Lai, K. S. & Lai, M. 1991 A cointegration test for market efficiency. *J. Futures Markets* **11**, 567–576.
- Lee, T.-H. 1994 Spread and volatility in spot and forward exchange rates. *J. Int. Money Finance* **13**, 375–383.
- Lo, A. W. & MacKinlay, C. A. 1988 Stock market prices do not follow random walks: evidence from a simple specification test. *Rev. Financial Studies* **1**, 41–66.
- MacDonald, R. & Taylor, M. 1988 Metals prices, efficiency and cointegration: some evidence from the London Metal Exchange. *Bulletin Econ. Res.* **40**, 235–239.
- MacDonald, R. & Taylor, M. P. 1994 The monetary model of the exchange rate: long-run relationships, short-run dynamics and how to beat a random walk. *J. Int. Money Finance* **13**, 276–290.
- Mackinnon, J. G. 1994 Approximate asymptotic distribution functions for unit-root and cointegration tests. *J. Business Econ. Statist.* **12**, 167–176.
- Masih, R. 1997 Cointegration of markets since the '87 crash. *Q. Rev. Econ. Finance* **37**.
- Nieuwland, F. G. M., Verschoor, W., Willen, F. C. & Wolff, C. C. P. 1994 Stochastic trends and jumps in EMS exchange rates. *J. Int. Money Finance* **13**, 669–727.
- Nugent, J. 1990 Further evidence of forward exchange market efficiency: an application of cointegration using German and UK data. *Econ. Social Rev.* **22** 35–42.
- Phillips, P. C. B. & Perron, P. 1988 Testing for a unit root in time series regressions. *Biometrika* **75**, 335–346.
- Pindyck, R. S. & Rothenberg, J. J. 1992 The comovement of stock prices. *Q. J. Econ.* 1073–1103.
- Proietti, T. 1997 Short-run dynamics in cointegrated systems. *Oxford Bull. Econ. Statist.* **59**.
- Phil. Trans. R. Soc. Lond. A* (1999)

- Schmidt, P. & Phillips, P. C. B. 1992 LM tests for a unit root in the presence of deterministic trends. *Oxford Bull. Econ. Statist.* **54**, 257–288.
- Schroeder, T. C. & Goodwin, B. K. 1991 Price discovery and cointegration for live hogs. *J. Futures Markets* **11**, 685–696.
- Schwarz, T. V. & Laatsch, F. E. 1991 Dynamic efficiency and price discovery leadership in stock index cash and futures market. *J. Futures Markets* **11**, 669–684.
- Schwarz, T. V. & Szakmary, A. C. 1994 Price discovery in petroleum markets: arbitrage, cointegration, and the time interval of analysis. *J. Futures Markets* **14**, 147–167.
- Smith, K. L., Brocato, J. & Rogers, J. E. 1993 Regularities in the data between major equity markets: evidence from Granger causality tests. *Appl. Financial Econ.* **3**, 55–60.
- Wang, G. H. K. & Yau, J. 1994 A time series approach to testing for market linkage: unit root and cointegration tests. *J. Futures Markets* **14**.